Bourgin-Yang version of the Borsuk-Ulam theorem

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October 13, 2012

Abstract

The classic Borsuk-Ulam theorem states that for $n > m$ there are no maps $f : \mathbb{S}^n \to \mathbb{S}^m$ such that $f(-x) = -f(x)$. This result has various generalizations, for instance, requiring more complicated symmetries of f. It has also many applications from analysis to graph theory.

In 1954 and 1955 C. T. Yang and (independently) D. G. Bourgin proved a theorem on $\mathbb{Z}_2$-equivariant maps f from the unit sphere $S(\mathbb{R}^n) \subset \mathbb{R}^n$ into $\mathbb{R}^m$, where the Euclidean spaces are considered as representations of $\mathbb{Z}_2$ with the antipodal action. They showed that for the set of zeroes

$$\mathbb{Z}_f \overset{\mathtt{def}}{=} f^{-1}(0)$$

we have the estimate

$$\dim Z_f \geq n - m - 1$$

where dim denotes the covering dimension.

In this talk we present corresponding theorems for G-equivariant maps with G equal to $\mathbb{Z}_{p^k}$ (a prime power cyclic group), $(\mathbb{Z}_p)^k$ (a p-torus), and to $\mathbb{T}^k$ (the k-dimensional torus).